Work

Work is done whenever a force acts to displace something.

$$W = \overrightarrow{F}.\overrightarrow{dl}$$

We say work has been done by force $\overrightarrow{F}$ on the thing displaced.

It has units of energy i.e. Joules.

Consider a system in thermodynamic equilibrium. Equilibrium means there is no macroscopic motion present in the system. In particular, the boundaries of the system are not moving as a consequence of mechanical equilibrium.
Now suppose some force acts on the system for example, displaces its boundaries, then this force has done some work on the system.
Take the example of a gas trapped in a box, fitted with a frictionless piston on top.There is some weight sitting on top of the piston say 5 kg. Let us assume that the gas is in equilibrium, i.e. the piston is not moving. That means the force of pressure by the gas on the piston, pushing it outwards is being balanced by the downward force of weight due to gravity.

Now if we add another 5 kg on top, the forces will no longer be in equilibrium and the piston will move downward. This means that "environment"(weight being outside of the system) has done some work on the system. if the piston moves by a distance 'l', before coming to stop again , the work done by environment is equal to

$$(5 + 5) \times gN \times l$$

where 'g' is acceleration due to gravity

Quasi-static Work:

Adding the weight on top brings the system out of equilibrium. Infact it was this very fact that caused the piston to move. the system does come back to equilibrium. This means we can not represent the system via state variables 'during' the process. For example, when weight is suddenly increased on top, there will be turbulence produced inside the gas and there will be no well defined value of the pressure throughout the system.

Many a time it is desirable to calculate certain quantities , like work done on the system, in terms of the variables of the system itself. How can we do it if the variables are not even defined during the process?

We consider what are called " Quasi-static Processes". A quasi-static process is such a change in the state of the system that the system more or less remains in equilibrium throughout the process. For example quasi-static work can be done by slightly changing the external force, so that the system remains in equilibrium more or less, at any given instant. So we increase the force very slowly so that the system never really goes out of equilibrium, but still it's variables , for example pressure adjust to the increase in the external force and change.

An ideal quasi-static process would ofcourse be infinitely slow. But we can approximate slow enough processes by quasi-static process.
A very important point to note is that Thermodynamics concerns itself mostly with just producing the states that will be rendered during a process.
Let us come back to our example of gas in the container. We can do quasi-static work, by adding slight amounts of grams on top, so that the weight increase is very small. At each step, the pressure of the gas will only slightly increase, and the turbulence etc, will be so small that they can be neglected. Hence for all practical purposes the system remains in equilibrium throughout.
In this case we can represent the work done via state variables of the gas.
By defination, the forces on the boundary are always balanced during a quasi-static process. Hence, the work done on gas can be calculated using pressure of the gas as well.
Suppose the piston moves up a little, then change in volume is positive. If the area of piston is A, the distance it moved dl is

$$\text{dV=Adl}$$

Then the work done "by the gas" is

$$\overrightarrow{F_{\text{gas}}}.\overrightarrow{\text{dl}} = \text{P.Adl} = \text{PdV}$$

And the Work done "on the gas" by the external Force is

$$\text{Work on gas = -PdV}$$

Negative sign appears because the external force is directed opposite to the direction of movement.
Because of equality of external force and internal pressure, the work done by the environment on the system, and the work done by the system on the environment is always equal and opposite during quasi-static process.
Quasi-static processes usually give the most useful works and can be thought of as no friction idealiztion in ususal mechanics.
For quasi-static process,

$$\text{dW=-PdV where P is pressure of gas.}$$

And the total Work done in going from state 1 to state 2 is

$$\int_1^2 dW = W_{1\longrightarrow 2} = -\int_{V_1}^{V_2} PdV \tag{1}$$

where V_1 and V_2 are the volumes of the gas at state 1 and 2 respectively.
Because the system remains in equilibrium during the process, we can use equation of state at each step of the process. We an also describe the changes in the system by a

path on the phase diagram.

Quasi-static Work:

Let us calculate work done on a gas in such a way that its temperature does not change during the [text missing]

We have

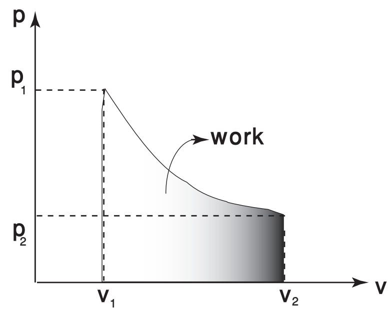
$$W = -\int_1^2 PdV = -\int_1^2 \frac{nRT}{V}.dV$$

where we have used $PV = nRT$

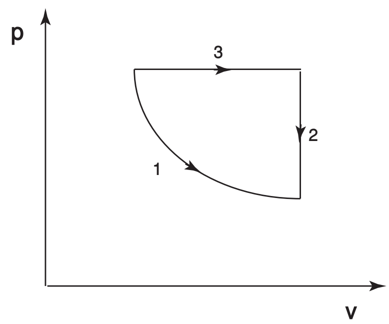
$$\begin{aligned} W &= -nRT \int_1^2 \frac{1}{V}.dV \\ &= -nRT\ log\ V|_{V_1}^{V_2} \\ &= -nRT\ log\ \frac{V_2}{V_1} \end{aligned}$$

If $V_2 < V_1$, this work is positive and if $V_2 > V_1$, this work is negative. If we describe any process on a P-V diagram, then the work done on the system is just the area under the curve representing the process.

$$\int_1^2 PdV = \text{Area under the curve.}$$



Notice that the work done on a system depends on path. We can start at same state (P_1, V_1) and end up at a given state (P_2, V_2), but the work done may be different depending on what path we take. For example the Work done during the two processes shown in figure is not same, although the initial and final states are same[text missing]



It is also clear that work is not a state variable. Clearly, it depends on path taken.