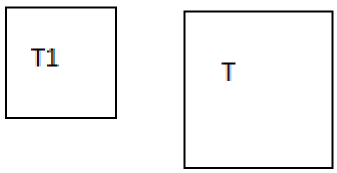
PHY 202 Heat and Thermodynamics **Recitation#4**

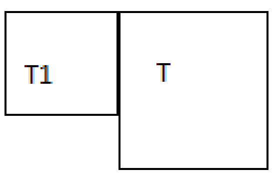
Q#1



Consider a body at temperature T1 and a reservoir at temperature T.
(a) Calculate the change in entropy of the body and reservoir seperately when they are brought together?
(b) Show that the total change in entropy of the universe is positive.
Solution:
(a) For the reservoir :

$$\triangle \mathbf{S}_{reservoir} = \oint dQ_{rev}/T$$

The two bodies (reservoir and body) are brought together and maintained at a temperature 'T' (since the reservoir has huge heat compared to the body so it can maintain its temperature).



Entropy defined for this reversible process: ($Q = m * C_v * \triangle T$ or $Q = C_v * \triangle T$)

$$\triangle \mathbf{S}_{reservoir} = -(C_v * \triangle T)/T$$
heat given off by reservoir = heat taken in by the body
$$Q = C_v(T - T1)$$
$$\triangle \mathbf{S}_{reservoir} = C_v(T1 - T)/T$$

Similarly, Entropy of the body is given by:

$$\triangle \mathbf{S}_{body} = \int_{T1}^{T} dQ_{rev}/T$$
$$\triangle \mathbf{S}_{body} = \int_{T1}^{T} C_v * dT/T$$
$$= C_v ln(T/T1)$$

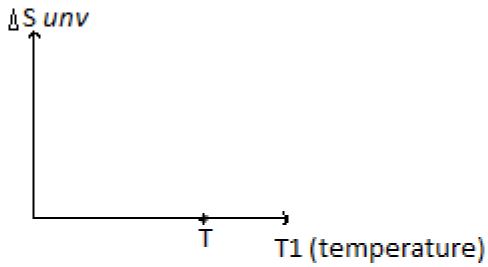
(b)

$$\text{Total change in entropy of the universe} = \triangle\mathbf{S}_{body} + \triangle\mathbf{S}_{reservoir}$$
$$= C_v ln(T/T1) + C_v(T1 - T)/T$$
$$= C_v(ln(T/T1) + T1/T - 1)$$

To prove that above is a positive value, lets first consider the worst case scenario where T=T1

$$\triangle \mathbf{S}_{universe} = C_v(ln(T/T) + T/T - 1)$$
$$= 0$$

Therefore, at T=T1, $\triangle \mathbf{S}_{universe}$ will be zero. In order to prove that $\triangle \mathbf{S}_{universe}$ is a positive quantity , we need to prove that $\triangle \mathbf{S}_{universe}$ stays above the temperature axis in the graph below:



$d^2S_{universe}/dT1^2$ should be negative in the region where T1 < T for the graph to stay in the positive half of the $\triangle \mathbf{S}_{universe}$ axis.

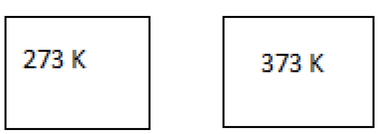
$$\triangle \mathbf{S}_{universe} = C_v(ln(T/T1) + T1/T - 1)$$
$$d/dT_1 \triangle \mathbf{S}_{universe} = ?$$
$$d/dT_1(\triangle \mathbf{S}_{universe}) = d/dT_1(ln(T/T1)) + 1/T$$

Computing the function:

$$1/T + d/dT_1(ln(T/T_1) = d/du(ln(u)) * du/dT_1$$
$$= \frac{T_1(d/dT_1(T/T_1)) + 1}{T}$$
$$= 1/T + T_1(-1/T_1^2)$$
$$= 1/T - 1/T_1$$
$$\frac{d(\triangle \mathbf{S}_{universe})}{dT_1} = \frac{1}{T} - \frac{1}{T_1}$$

for $T_1 < T, d/dT_1(\triangle S_{universe})$ will always be negative, hence proved.

Q # 2



(a) Two bodies shown above are brought together, Calculate the entropy change of each body? (assume masses are equal) $C_v of both = 1800 J/K$
(b) Show that the total entropy change is positive.
Solution:
(a) Entropy change of body A: $\triangle S_A = \int dQ_{rev}/T$
When they are brought together , the final temperature of both is :
Heat gained by body A = Heat lost by body B
assume final temperature of both is T

$$dQ = C \triangle T$$
$$dQ_A = dQ_B$$
$$C_v(T - 273) = -C_v(T - 373)$$
$$\text{so, } T = 323K.$$

At T=323 K

$$\triangle S_A = \int_{273}^{323} dQ_{rev}/T$$
$$\triangle S_A = C_v \int_{273}^{323} dT/T$$
$$= C_v ln(323/273)$$
$$= 1800 * ln(323/273)$$
$$= 302.72 J/K$$

For body B:

$$\triangle S_B = \int_{373}^{323} dQ_{rev}/T$$
$$\triangle S_B = C_v \int_{373}^{323} dT/T$$
$$= C_v ln(323/373)$$
$$= 1800 * ln(323/373)$$
$$= -259.07 J/K$$

(b) Total entropy change of universe $= \triangle S_A + \triangle S_B$
$= 302.72 + (-259.01)$
$= 43.65 J/K$

Hence, the entropy change of the universe is positive. Also, note that the decrease in entropy of the body at a higher temperature is always less than the increase in entropy for the cooler body, hence the entropy change for the universe will always come out to be positive.