PH-202 **Solution Recitation #2** February, 15, 2011

Answer 1:

Differential form of first law of thermodynamics is,

$$dU = dQ + dW.$$

Since vessel is made of adiabatic walls, so it is an isolated system, and for an isolated system, $dQ = 0$, $\Rightarrow dU = 0$ and $dT = 0$. In general, the internal energy of any gas is a function of any two of the coordinates P, V and T. The differential of U as a function of T and V is,

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV.$$

In the given case, no temperature change ($dT = 0$) takes place in the free expansion ($dU = 0$). Therefore,

$$\left(\frac{\partial U}{\partial V}\right)_T = 0.$$

or in other words, U does not depends on V. Now considering U to be a function of T and P, we have,

$$dU = \left(\frac{\partial U}{\partial T}\right)_P dT + \left(\frac{\partial U}{\partial P}\right)_T dP.$$

Again, if no temperature change ($dT = 0$) takes place in the free expansion ($dU = 0$). then it follows that,

$$\left(\frac{\partial U}{\partial P}\right)_T = 0.$$

or in other words, U does not depend on P. Then it is apparent that, if no temperature change takes place in a free expansion of a gas, U is independent of P and V, and therefore, U is a function of T only, i-e

$$U = U(T).$$

Answer 2:

Differential form of first law of thermodynamics is,

$$dU = dQ + dW.$$

As the process is adiabatic dQ = 0

$$dU = dW$$

Putting $dU = fndT$ and $dW = -PdV$ in above equation, gives us

$$fndT \;=\; -\,PdV \tag{1}$$

From the Ideal gas equation, we have

$$\begin{aligned} PV &= nRT \\ &\text{Differentiating both sides} \\ PdV \; + VdP &= nRdT \end{aligned}$$

$$\frac{PdV \;+\; VdP}{nR} \;=\; dT \tag{2}$$

Now taking this dT value and inserting it in (??)

$$\begin{aligned} fn(\frac{PdV \;+\; VdP}{nR}) &= -\,PdV \\ \frac{f}{R}(PdV \;+\; VdP) &= -\,PdV \\ \frac{f}{R}VdP &= -\,(1+\frac{f}{R})PdV \\ -\int \frac{dP}{P} &= (1+\frac{R}{f})\int \frac{dV}{V} \\ -lnP &= (1+\frac{R}{f})\; lnV \end{aligned}$$

where$(1+\frac{R}{f})$ is a constant which we are taking as γ.

$$\begin{aligned} -lnP &= \gamma\; lnV \\ -lnP &= lnV^{\gamma} \\ lnP \;+\; lnV^{\gamma} &= 0 \\ ln(PV^{\gamma}) &= 0 \end{aligned}$$

Taking exponential on both sides

$$PV^{\gamma} \;=\; \text{constant}$$

Answer 3:

Again taking differential form of first law of thermodynamics,

$$dU = dQ + dW.$$

Where, $W = -PdV$,

$$\Rightarrow dU = dQ - PdV. \tag{3}$$

for constant volume the heat capacity is given by,

$$C_V = \left(\frac{dQ}{dT}\right)_V.$$

As volume is constant so dW = 0, then

$$dU = C_V dT. \tag{4}$$

Now, for constant pressure again write the first law of thermodynamics

$$dU = dQ + dW.$$

and substituting $dW = -PdV$ the value of dU from equation (**??**) yields,

$$\begin{aligned} C_V dT &= dQ - PdV, \\ dQ &= C_V dT + PdV. \end{aligned} \tag{5}$$

Now consider ideal gas equation,

$$PV = nRT.$$

and, for an infinitesimal process,

$$\begin{aligned} PdV + VdP &= nRdT, \\ PdV &= nRdT - VdP. \end{aligned}$$

Substituting the value of PdV in equation (**??**), we get,

$$\begin{aligned} dQ &= C_V dT + (nRdT - VdP), \\ dQ &= (C_V + nR)dT - VdP. \end{aligned} \tag{6}$$

and dividing by dT yields,

$$\frac{dQ}{dT} = C_V + nR - V\frac{dP}{dT}.$$

At constant pressure, the left hand side is equal to C_P and $dP = 0$, therefore,

$$\begin{aligned} C_P &= C_V + nR, \\ \text{or} \quad \frac{C_P}{C_V} &= 1 + \frac{nR}{C_V}. \end{aligned} \tag{7}$$

We know that internal energy is directly proportional to temperature,

$$\begin{aligned} U &= nfT, \\ \Rightarrow C_V &= \frac{dU}{dT} = nf. \qquad \text{from eq:4} \end{aligned}$$

Thus equation (**??**) will become,

$$\begin{aligned}\frac{C_P}{C_V} &= 1+\frac{nR}{nf}, \\ &= 1+\frac{R}{f}\end{aligned}$$

Where, $1+\frac{R}{f} = constant = \gamma$, Hence,

$$\frac{C_P}{C_V} = \gamma.$$