PH-202 **Solution Set # 2** February, 19, 2010

Answer 1:

(a) Entropy of water is given by,

$$\begin{aligned}
\Delta S_w &= \int_{T_1}^{T_2} mC_V \frac{dT}{T}, \\
&= mC_V \int_{T_1}^{T_2} \frac{dT}{T}, \\
&= mC_V |\ln T|_{T_1}^{T_2}, \\
&= mC_V \ln\left(\frac{T_2}{T_1}\right).
\end{aligned}$$

Substitution of the given data yields,

$$\begin{aligned}
\Delta S_w &= 1 \times 4187 \times \ln\left(\frac{373}{273}\right), \\
&= 1306.8 \ J/K
\end{aligned}$$

Entropy of hot reservoir is given by,

$$\Delta S_r = -\frac{Q}{T},$$

Where, Q = Heat lost by reservoir = Heat gained by water = $mC_V \Delta T$,

$$\begin{aligned}
\Delta S_r &= -\frac{mC_V \Delta T}{T}, \\
&= -\frac{1 \times 4187 \times (373 - 273)}{373}, \\
&= -1122.5 \ J/K.
\end{aligned}$$

Entropy of universe will be,

$$\begin{aligned}
\Delta S_u &= \Delta S_w + \Delta S_r, \\
&= 1306.8 - 1122.5 = 184 \ J/K.
\end{aligned}$$

(b) Since entropy is a state function, it doesn't depend on path. Thus change in entropy of water is $\Delta S_w = 1306.8 \ J/K$ as calculated in part(a). When water at $273^o K$ is brought in contact with reservoir at temperature $323^o K$, change in entropy of reservoir will be,

$$\begin{aligned}
\Delta S_1 &= -\frac{Q_1}{T} = -\frac{mC_V \Delta T}{T}, \\
&= -\frac{1 \times 4187 \times (323 - 273)}{323} = -648 \ J/K.
\end{aligned}$$

When water is brought in contact with reservoir at temperature $373^o K$, change in entropy will be,

$$\Delta S_2 = -\frac{1 \times 4187 \times (373 - 323)}{373} = -561.26\ J/K.$$

change in entropy of universe will be,

$$\begin{aligned} \Delta S &= \Delta S_w - \Delta S_1 - \Delta S_2, \\ &= 1306.8 - 648 - 561.26 = 97.54\ J/K. \end{aligned}$$

(c) From part (a) of this question we see that, entropy of the universe is 184 J/K if temperature of the water rises from $273K$ to $373K$, only in one step. While from part (b), it is clear that, if water is heated in two steps i.e. from $273K$ to $323K$ and then from $323K$ to $373K$, change in entropy of the universe is 97.54 J/K, which is smaller than the entropy in part (a).
This shows that if we raise the temperature of the water in more and more small steps entropy of the universe decreases, and we can make it approximately zero by using an infinite number of reservoirs to raise the temperature of the water in infinitesimal small amount.

Answer 2:

(a) Entropy change of copper block is,

$$\begin{aligned} \Delta S_c &= \int_{T_1}^{T_2} C\frac{dT}{T}, \\ &= C\int_{T_1}^{T_2} \frac{dT}{T}, \\ &= C|\ln T|_{T_1}^{T_2}, \\ &= C\ln\left(\frac{T_2}{T_1}\right). \end{aligned}$$

Substitution of given data yields,

$$\begin{aligned} \Delta S_c &= 150 \times \ln\left(\frac{283}{373}\right), \\ &= -41.42\ J/K. \end{aligned}$$

Entropy change of lake will be,

$$\begin{aligned} \Delta S_l &= \frac{Q}{T} = \frac{C\Delta T}{T}, \\ &= \frac{150 \times (373 - 283)}{283}, \\ &= 47.07\ J/K. \end{aligned}$$

Entropy change of universe will be,

$$\begin{aligned}\Delta S_u &= \Delta S_c + \Delta S_l,\\ &= -41.42 + 47.07 = 5.65\ J/K.\end{aligned}$$

(b) Since the block is thrown from a height of 100 m, at the top, block will have P.E. While striking at the water surface its P.E will completely change into heat. i.e,

$$\begin{aligned}P.E &= Q = mgh,\\ Q &= 0.4 \times 10 \times 100 = 400\ J.\end{aligned}$$

This heat will transfer to the lake. Thus change in entropy of lake will be,

$$\Delta S_l = \frac{Q}{T} = \frac{400}{283} = 1.39\ J/K.$$

Since no heat enters the block nor does its state change in any manner (staying at the same temperature and retaining same volume etc.$\Delta S_b = 0$. Thus entropy change of the universe will be,

$$\begin{aligned}\Delta S_u &= \Delta S_l + \Delta S_b,\\ &= 1.39\ J/K\end{aligned}$$

(c) We are given that,
Temperature of 1st block= $100^oC = 373\ K$,
Temperature of 2nd block= $0^oC = 273\ K$,
When two blocks are joined together heat transfers from 1st block to 2nd block. Then according to law of heat exchange,

$$\begin{aligned}\text{Heat lost by 1st block} &= \text{Heat gained by 2 block},\\ \Delta Q_1 &= \Delta Q_2,\\ mC\Delta T_1 &= mC\Delta T_2,\\ \Delta T_1 &= \Delta T_2,\\ (373 - T_f) &= (T_f - 273),\\ T_f &= 323\ K.\end{aligned}$$

Now change in entropy of 1st block will be,

$$\begin{aligned}\Delta S_1 &= mC\ln\left(\frac{T_f}{T_i}\right),\\ &= 0.4 \times 150 \times \ln\left(\frac{323}{373}\right),\\ &= -8.64\ J/K.\end{aligned}$$

Similarly change in entropy of 2nd block will be,

$$\begin{aligned}\Delta S_2 &= mC\ln\left(\frac{T_f}{T_i}\right), \\ &= 0.4\times 150\times \ln\left(\frac{323}{273}\right), \\ &= 10.09\ J/K.\end{aligned}$$

Thus entropy change of university will be,

$$\begin{aligned}\Delta S_u &= \Delta S_1 + \Delta S_2, \\ &= 1.45\ J/K\end{aligned}$$

Answer 3:

a)As the resistor remains at the same temperature and none of its other properties are changing with time, its state is not changing. Hence Entropy being a state variable remains same. b)We have just seen that the state of the system remains same, therefore $dE = 0$ and according to first law

$$dW = -dQ$$

the work done by the passage of current through the resistance is converted into heat and it is being released into the surroundings constantly. This heat 'Q' can be calculated by the following formula,

$$\begin{aligned}Q &= I^2 \times R \times t \\ &= 10^2 \times 25 \times 1 \\ &= 2500J \\ \Delta S_{surrounding} &= \frac{Q}{T} \\ &= \frac{2500}{273+27} \\ &= 8.33J/K\end{aligned}$$

What we need to calculate is$\Delta S_{universe}$, which is equal to

$$\begin{aligned}\Delta S_{universe} &= \Delta S_{surrounding} + \Delta S_{system}, \\ &= 0 + 8.33 \\ &= 8.33J/K\end{aligned}$$

c) This time all the work done by the system goes on to be converted into heat that is absorbed by the resistor itself. This will result in change in temperature of the resistor. So first we need to find the final temperature of the resistor.

Then we connect the initial state of the resistor with temperature 27^0C with the final state through an reversible heating process. The final temperature is found by equating the heat absorbed , which is just the heat capacity times the change in temperature with the work done by the current i.e.,

$$C(T_f - T_i) = I^2 \times R \times t$$
$$T_f = 2500J/C + 27^0C$$

Now here for C you have to put the heat capacity of $10g$ of copper to find out the final temperature. (It is 10 times the specific heat capacity per gram).

Then you use the formula

$$\Delta S = \int_{T_i}^{T_F} C\frac{dT}{T}$$

to find the change in entropy of the resistor.

d) Since there is no change in anything in the surroundings, as our system was insulated, the change in entropy of the universe is just equal to the change in entropy of the resistor in part c.

Answer 4:

We have two reservoirs T_1 and T_2 exchanging heat where$T_1 > T_2$. Heat exchange is occurring through a rod whose state remains same i.e $\Delta S = 0$.
Let the heat flow from T_1 to T_2 be Q then

$$\Delta S_{T_1} = \int \frac{dQ}{T_1} = -\frac{Q}{T_1}$$

The minus sign comes here because the heat is going out through this system. Similarly,

$$\Delta S_{T_2} = \int \frac{dQ}{T_2} = \frac{Q}{T_2}$$

Here the heat is coming in so Q is positive.
Then, the change in the entropy of universe is

$$\begin{aligned}\Delta S_{universe} &= -\frac{Q}{T_1} + \frac{Q}{T_2} \\ &= Q\frac{T_1 - T_2}{T_1 T_2}\end{aligned}$$

Answer 5:

According to second law of thermodynamics, change in entropy is either positive or zero. If we take the above case again but now let us consider that $T_2 > T_1$. The heat 'Q' is flowing from T_1 to T_2 i.e from a cold reservoir to a hotter one, violating the kelvin's statement.

Then as we computed in the last question entropy of universe is given by

$$\Delta S_{universe} = Q\frac{T_1 - T_2}{T_1 T_2}$$

here as you can see $\Delta S_{universe} < 0$ which is against the second law of thermodynamics. Thus violating kelvin's statement we end up with violating the 2nd law thus kelvin statement should hold.