PH-202 **Solution Set # 2** March, 01, 2011

Solution 1:

We know that $P = \rho g h$ then

$$\frac{\partial P}{\partial h} = -\rho g$$

- sign comes here because pressure decreases with height.
We also have $\rho = \frac{PM}{RT}$ from ideal gas equation , then

$$\frac{\partial P}{\partial h} = -\frac{PM}{RT}g$$

then readjusting we have

$$\frac{\partial P}{P} = -\frac{Mg}{RT}\partial h$$

Now we are given equation

$$PV^{\gamma} = const$$

From ideal gas equation PV=nRT we have

$$V = \frac{nRT}{P}$$

Putting in value of V in above equation we have

$$P^{1-\gamma}(nRT)^{\gamma} = const$$
$$P^{1-\gamma}T^{\gamma} = const$$

Differentiating both sides and adjusting we have

$$\frac{\partial P}{P} = -\frac{\partial T}{T}\frac{\gamma}{1-\gamma}$$

Equating above two equations, we get

$$-\frac{\gamma}{1-\gamma}\frac{\partial T}{T} = -\frac{Mg}{RT}\partial h$$

Rearranging we have

$$\frac{\partial T}{\partial h} = \frac{Mg}{R}\frac{1-\gamma}{\gamma}$$

This is our required differential equation. b) Now solving it we have

$$T - T_0 = \frac{1-\gamma}{\gamma}\frac{Mgh}{R}$$

Putting in h= $40,000ft = 12192m, \gamma = 1.4, R = 8.31J/KmolM = .0286g/mol$ we have

$$\begin{aligned} T &= T_0 - 119.08 \\ T &= 178.91K \end{aligned}$$

Solution 2:

a) We know that

$$\begin{aligned} Q &= mC\Delta T \\ &= .25 \times 4187 \times (24 - 20) \\ &= 4187J. \end{aligned}$$

b)Heat gained by water = Heat lost by the metal.
c) Heat capacity of metal can also be calculated from the above formula,

$$\begin{aligned} Q &= C\Delta T \\ C &= \frac{Q}{\Delta T} \\ &= \frac{4187}{100 - 24} \\ &= 1046.75J/C \end{aligned}$$

Solution 3:

We have given that,
Heat capacity of pasta= $C_p = 1.8\ J/C$,
Mass of pasta= $m_p = 400\ g$,
Initial temperature of pasta= $(T_i)_p = 25C^o$,
Heat capacity of water= $C_w = 4187\ J/C$,
Mass of water= $m_w = 1.5\ lit$,
Initial temperature of water= $(T_i)_w = 100C^o$,
We want to calculate the final temperature T_f of water and pasta. Consider combined system of pasta and water and no heat exchange with surrounding i.e. $dU = 0 \ \Rightarrow dW = 0$, then according to first law of thermodynamics,

$$\begin{aligned} dU &= dQ + dW, \\ dU &= (Q_p + Q_w) + dW, \end{aligned}$$

Where Q_p and Q_w are the heat of pasta and water respectively. Since dU and dW both are zero, therefore,

$$\begin{aligned} Q_w &= -Q_p, \\ m_w C_w \Delta T_w &= -m_p C_p \Delta T_p, \\ m_w C_w (T_f - (T_i)_w) &= -m_p C_p (T_f - (T_i)_p), \end{aligned}$$

Substitution of the given data yields,

$$T_f = 92.3^o C.$$

Solution 4:

a) Let's consider orbit of one particle around the other massive and stationary particle.
For a particle moving in a circle, centrifugal force is given by

$$F = \frac{mv^2}{r}$$

This force is balanced by the gravitational attraction force

$$F = G\frac{mM}{r^2}$$

this gives us

$$\frac{1}{2}mv^2 = G\frac{mM}{2r}$$

This is the kinetic energy of the smaller particle. Now the second massive particle is stationary so it has zero K.E.
Potential energy can be calculated from the gravitational attraction force using

$$F = -\frac{\partial U}{\partial r}$$

which gives us by integration

$$U = -\frac{GmM}{r}$$

Comparing with the above kinetic energy equation, we have

$$K.E = -2P.E$$

b) Adding some energy to the system, increases the particle's velocity and it's orbit increases, decreasing its K.E since K.E = gravitational attraction force.
c) Heat capacity is given by

$$\begin{aligned} C &= \frac{Q}{\Delta T} \\ E_T &= E_{K.E} + E_{P.E} \end{aligned}$$

From virial theorem we have,
P.E = -2K.E

$$E_T = -2E_{K.E} + E_{K.E}$$

and as $E_{K.E} = nRT$

$$E_T = -nRT$$

then C = -nR. It's negative.

Solution 5:

We had derived the following expression in recitation

$$\eta = 1 + \frac{T_3 ln\frac{V_4}{V_3}}{T_1 ln\frac{V_2}{V_1}}$$

For adiabatic compression at point 3 we have

$$P_3 V_3^\gamma = const$$

which gives

$$nRT_3V_3^{\gamma-1} = const$$

Similarly we can calculate

$$\begin{aligned} nRT_1V_1^{\gamma-1} &= const \\ nRT_2V_2^{\gamma-1} &= const \\ nRT_4V_4^{\gamma-1} &= const \end{aligned}$$

Now since$T_1 = T_2$ and $T_3 = T_4$, above equation can be compared to give

$$\frac{V_3}{V_4} = \frac{V_2}{V_1}$$

which leads to

$$\eta = 1 - \frac{T_3}{T_1}$$