PH-202 **Solution Set # 1** February, 9, 2011

Answer 1:

(a) State equation for thermal equilibrium between systems A and C is,

$$P_AV_A - P_CV_C - nbP_A = 0 \tag{1}$$

State equation for thermal equilibrium between systems B and C is,

$$P_BV_B - P_CV_C + nB\left(\frac{P_CV_C}{V_B}\right) = 0 \tag{2}$$

From equation (1), we have,

$$P_CV_C = P_AV_A - nbP_A \tag{3}$$

Similarly from equation (2),

$$P_BV_B + P_CV_C\left(\frac{nB}{V_B} - 1\right) = 0$$
$$P_CV_C = P_BV_B\left(\frac{V_B}{V_B - nB}\right) \tag{4}$$

Comparing equation (3) and equation (4), we have,

$$P_AV_A - nbP_A = P_BV_B\left(\frac{V_B}{V_B - nB}\right) = P_CV_C = \theta$$

Since all terms on one side are function of one system, and they are all equal, hence the empirical temperatures are,

$$\begin{aligned}
\theta_A &= P_AV_A - nbP_A \\
\theta_B &= P_BV_B\left(\frac{V_B}{V_B - nB}\right) \\
\theta_C &= P_CV_C
\end{aligned}$$

(b) In thermal equilibrium, θ_A and θ_B are equal in value, giving the relation,

$$P_AV_A - nbP_A = P_BV_B\left(\frac{V_B}{V_B - nB}\right)$$

<u>For system A</u>:

$$\begin{aligned}
P_AV_A - nbP_A &= T_E \\
P_AV_A &= T + nbP_A
\end{aligned}$$

<u>For system B</u>:

$$\frac{P_BV_B}{\frac{V_B - nb}{V_B}} = T_B$$

<u>For system C</u>:

$$P_CV_C = T_C$$

Answer 2:

a) We are going to use the expression

$$P = P_0 \exp(\frac{-Mgz}{RT})$$

where M = molar mass of air = 0.0289 kg/mol
R = ideal gas constant = 8.31 Nm/(mol.K)
T = 298.15 K

$$\begin{aligned} P &= P_0 \exp(\frac{-z}{8.74km}) \\ \frac{P}{P_0} &= \exp(\frac{-z}{8.74km}) \end{aligned}$$

i) when pressure is10% of its value at sea level:

$$0.1 = \exp \frac{-z}{8.74km}$$

taking ln on both sides

$$\begin{aligned} ln(0.1) &= \frac{-z}{8.74km} \\ z &= -8.74(-2.302) \\ &= 20.11km \end{aligned}$$

ii) When pressure is 1% of its value at sea level

$$\begin{aligned} ln(0.01) &= \frac{-z}{8.74km} \\ z &= 40.17km \end{aligned}$$

b) We know

$$\begin{aligned} PV &= nRT \\ \frac{P}{RT} &= \frac{n}{V} = \frac{\rho}{m} \end{aligned}$$

as T, R and m are constants then P $\infty \rho$.
if pressure is 10% of it's value then density is also. Air density decreases with altitude as pressure.

Density of air at sea level = $1.29 kg/m^3$

$$\begin{aligned} \text{Density of air at height 18.42 (} P &= 10\% P_0 \text{)} = .1 \times 1.29 = .129 \ kg/m^3 \\ \text{Density of air at height 36.84 (} P &= 1\% P_0 \text{)} = .01 \times 1.29 = .0129 \ kg/m^3 \end{aligned}$$

c) Air pressure at the top of mount everest is again calculated from

$$P = P_0 \exp \frac{-z}{8}$$

here z = height of mount everest = 8.8 km
Pressure at sea level = P_0 = 1 atm

$$\begin{aligned} P &= 1 \times \exp \frac{-8.8}{8} \\ &= 0.33 \; atm \end{aligned}$$

Pressure at 40,000 ft (12.192 km):

$$\begin{aligned} P &= 1 \times \exp \frac{-12.192}{8} \\ &= .217 \; atm \end{aligned}$$

Air plane cabins have to be pressurized because the air pressure is very low at that altitude.

Force per unit area on the cabin pushing outwards is equal to the difference in the inside and outside air pressure . As inside air pressure is maintained at 0.85 atm and the outer pressure is calculated to be .217atm, the value of force per unit area is .85 - .217 = 0.633 atm. If the pressure difference is increased more than this than as a result force pushing outwards on cabin walls increases which obviously is bad for aeroplane structure.

Answer 3:

Van-der Wall equation of state is,

$$\begin{aligned} \left(P + \frac{n^2 a}{V^2}\right)(V - nb) &= nRT \\ \Rightarrow \left(P + \frac{n^2 a}{V^2}\right) &= \frac{nRT}{V - nb} \\ P &= \left(\frac{nRT}{V - nb} - \frac{n^2 a}{V^2}\right) \end{aligned}$$

Work done in compressing a gas isothermally is given by,

$$W = -\int_{V_1}^{V_2} P dV \tag{5}$$

By substituting value of P in equation (5), we obtained,

$$\begin{aligned}
W &= -\int_{V_1}^{V_2}\left(\frac{nRT}{V-nb}-\frac{n^2a}{V^2}\right)dV \\
&= -\int_{V_1}^{V_2}\left(\frac{nRT}{V-nb}\right)dV+\int_{V_1}^{V_2}\left(\frac{n^2a}{V^2}\right)dV \\
&= -nRT\int_{V_1}^{V_2}\left(\frac{1}{V-nb}\right)dV+n^2a\int_{V_1}^{V_2}\left(\frac{1}{V^2}\right)dV \\
&= -nRT\ln\Big|(V-nb)\Big|_{V_1}^{V_2}+n^2a\Big|-\frac{1}{V}\Big|_{V_1}^{V_2} \\
&= -nRT\ln\left(\frac{V_2-nb}{V_1-nb}\right)+n^2a\left(-\frac{1}{V_2}+\frac{1}{V_1}\right) \\
&= nRT\ln\left(\frac{V_1-nb}{V_2-nb}\right)+n^2a\left(\frac{1}{V_1}-\frac{1}{V_2}\right)
\end{aligned}$$

Above last equation is the required work done. Note that both the terms are positive.

Answer 4:

Equation of state for electromagnetic field in a box (photon gas) is,

$$P=\frac{1}{3}\sigma T^4$$

Where σ is a constant. **Along path a:**

Since work done is given by,

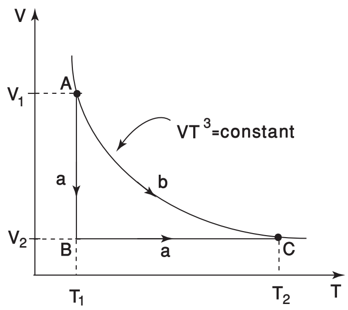

Figure 1: Experimental setup.

$$W=-\int_{V_1}^{V_2}PdV$$

There are two parts of path a i-e AB and BC. Along the first part AB temperature is constant, which implies that pressure is also constant. Thus work done along this part will be,

$$
\begin{aligned}
W_{AB} &= -\int_{V_1}^{V_2} \frac{1}{3}\sigma T^4 dV \\
&= -\frac{1}{3}\sigma T^4 \int_{V_1}^{V_2} dV \\
&= -\frac{1}{3}\sigma T^4 |V|_{V_1}^{V_2} \\
&= -\frac{1}{3}\sigma T^4 (V_2 - V_1) \\
&= \frac{1}{3}\sigma T^4 (V_1 - V_2)
\end{aligned}
$$

Along second part BC, V is constant . So no work is done. i-a

$$W_{BC} = 0$$

Hence total work done is,

$$
\begin{aligned}
W_a &= W_{AB} + W_{BC} \\
W_a &= \frac{1}{3}\sigma T^4 (V_1 - V_2)
\end{aligned}
$$

Along path b:

From the figure it is clear that, VT^3 =constant.

$$W_b = -\int_{V_1}^{V_2} P dV \tag{6}$$

Where

$$P = \sigma T^4/3 \tag{7}$$

Also,

$$
\begin{aligned}
VT^3 &= \text{constant} = K \\
\Rightarrow T &= \frac{K}{V^{1/3}}
\end{aligned}
$$

By substituting this value of T in equation 7, we get,

$$P = \frac{\sigma K}{3V^{4/3}}$$

Thus work done along path b will be,

$$
\begin{aligned}
W_b &= -\int_{V_1}^{V_2} \frac{\sigma K}{3V^{4/3}} dV \\
&= -\frac{\sigma K}{3} \int_{V_1}^{V_2} V^{-4/3} dV \\
&= -\frac{\sigma K}{3} \left| \frac{-3}{V^{1/3}} \right|_{V_1}^{V_2} \\
&= \sigma K \left(\frac{1}{{V_2}^{1/3}} - \frac{1}{{V_1}^{1/3}} \right)
\end{aligned}
$$

Which is the required work done.