Recitation 2: Partial Differentiation

1-Consider a vessel made of adiabatic walls, containing two compartments with gas enclosed in only one of the compartments. When the door between the compartments is opened, the gas freely expands to fill the whole vessel. What is the change is internal energy? It is found experimentally that there is no change in temperature in this free expansion of the gas. Use this result and 1st law of thermodynamics to argue that internal energy U of the gas is a function of temperature only i.e.,

$U = U(T)$

In fact, it is known that the internal energy of an ideal gas is directly proportional to the temperature (and number of moles as U is an extensive quantity)

$U = fnT$

where f is a constant.

2-Prove that for a quasi-static adiabatic process,

$PV^{gamma} = constant,$

where gamma is also a constant. What is its value?

3-Heat capacity of a system is defined as the amount of heat required to raise the temperature of the system by unit amount (Or heat given out as the temperature is reduced by a unit amount i.e.,

$C = dQ/dT$

Notice that it is not a differential of a function , rather just a ratio of two small experimentally measured quantities. Unless, we have a specific process at hand, heat capacity as defined above can be anything for a system. It depends on the process through which the heat is provided. Therefore, we define two heat capacities, with a specified process, which always give the same number for a particular system, as they are functions of state functions only. These are

Heat capacity at constant volume = $C_V = (dQ/dT)_V$

Heat capacity at constant pressure = $C_P = (dQ/dT)_P$

Prove that the ratio of these two heat capacities is constant for ideal gas.

4- Exact and Inexact Differentials

Consider the two differentials

df=2x

Consider the figure below.

1) Find out the work done along each path.

2) What is the sign of the work in each case?

3) What is the total work done by the gas? Is it positive?

4) How is the final state of the gas different from initial state after going through the three processes?

5) What can you say about the changes in the state of the gas and the fact that it did some work during this cycle.