Problem Set 2: Heat and the laws of Thermodynamics

1. ## Adiabatic Atmosphere

   In last problem set, we modelled the atmosphere as isothermal. But this is not a very good approximation. Clearly, the temperature of the atmosphere is quite low as we go up in altitude. In this problem, we will try to capture this variation in temperature via convection.

   a) The idea is that pockets of air can move around. Since air is a bad conductor of heat we can take this process to be adiabatic. As the bubble rises up, its temperature decreases. Now at any point the pressure of the air bubble can be assumed to be equal to the pressure around it. Use this fact and the adiabatic equation of state

   $$PV^{\gamma} = constant$$

   to derive a differential equation describing variation of temperature with height. Here $\gamma = 1.4$, the value for air. In a static situation, we can now take this temperature of the bubble to be the actual temperature of the atmosphere.

   b) Find out the temperature at the height where commercial jets fly i.e., 40,000 feet. Does it come out to be about right?

2. ## Measuring Heat Capacities

   Heat capacity is an extensive quantity, the more substance you have, the more heat you need to raise its temperature. Also, if not much work is involved, such as for fairly incompressible fluids and solids, heat capacity is usually unambiguous as heat capacities at constant volume and pressure are almost same.

   To measure the heat capacity of an object, all you usually have to do is put it in thermal contact with another object whose heat capacity you know. As an example, suppose that a chunk of metal is immersed in boiling water (100^0C), then is quickly transferred into a Styrofoam cup containing $250g$ of water at 20^0C. After a minute or so, the temperature of the contents of the cup is 24^0C. Assume that during this time no significant energy is transferred between the contents of the cup and the surroundings. The heat capacity of the cup itself is negligible.

(a) How much heat is lost by the water?

(b) How much heat is gained by the metal?

(c) What is the heat capacity of this chunk of metal?

The heat capacity of 1 liter =1 kg of water is $4187 J/C$.

3. **Boiling Pasta**

The heat capacity of a particular brand of pasta is $1.8\frac{J}{C}$ for $1g$ of pasta. Suppose that you toss $400g$ of this pasta at room temperature ($25C$) into 1.5 liters of boiling water. What effect does this have on the temperature of the water? Assume that the heat capacity of 1 liter of water is 4187 $\frac{J}{C}$ for 1 liter of water, there is no gain or loss of heat to the surroundings

4. **Heat Capacities of Stars**

Heat capacities do not always have to be positive. That is to say, it is possible that you give off heat and the temperature rises as a result. So it is important to remember that temperature is not a measure of a body's contained energy or heat(contained heat actually means nothing). It is a measure of equilibrium and the thing that tells which way the heat will move spontaneously.

Heat capacities of stars or other gravitationally bound particles is negative. Let us model a star as a gas (collection of free particles).

a) First prove that for two particles that are orbiting in a circle about their center of mass due to the gravitational force, the potential energy of the system is -2 times the kinetic energy. This little exercise is just meant to help you believe in a more general result, which I now state. For any number of particles bound by gravitational force

$$< U >_{pot} = -2 < U >_{kin}$$

where $< U >_{pot}$ and $< U >_{kin}$ are total "average" potential and kinetic energies respectively of all the particles, about the center of mass. Average means, time average over a long time. This is called Virial theorem.

b) Now suppose you add some energy to the system, and wait for the system to equilibrate. Does the kinetic energy increases or decreases?

c) Now let us come to the thermodynamics. We can take the gas of the star to be an ideal gas as a first approximation, with one qualification. We have the equation of state for an ideal gas

$$E = 3/2nRT$$

But this holds in the absence of gravitational potential energy between the particles. So this comes only from kinetic energy. That means if we want to generalize it to the case of stars, we must take only the kinetic part to be equal to nRT. Use this fact and the Virial theorem to derive the heat capacity of a star. Is it positive or negative?

5. Carnot's Engine

In recitation, we derived an expression for the efficiency of an ideal gas Carnot cycle. Complete that work to show that it turns out to be

$$\eta = 1 - \frac{T_3}{T_1}$$

where T_1 and T_3 are respectively the temperatures at the upper and lower isothermal paths.

Also prove that no other engine in the world can be more efficient than a Carnot's engine.