Problem Set 1: Temperature, thermal equilibrium and work

1. ## Empirical temperatures

Consider three systems A, B and C with state functions P_A, V_A; P_B, V_B; and P_C, V_C respectively. When A and C are in thermal equilibrium, they satisfy the equation,

$$P_A V_A - P_C V_C - nbP_A = 0\,.$$

When B and C are in thermal equilibrium, their state functions satisfy,

$$P_B V_B - P_C V_C + nB\frac{P_C V_C}{V_B} = 0\,,$$

where n, B and b are constants.

(a) What are the three functions which are equals to one another at thermal equilibrium and each of which is equal to the empirical temperature?

(b) What is the relation between coordinates of A and B in thermal equilibrium?

(c) Write down equations of states for the three systems in terms of empirical temperatures you just defined.

2. ## Isothermal Atmosphere

In class we derived an expression for variation of pressure with height above the sea level, modelling the atmosphere to be isothermal i.e., at same temperature throughout.

a) Put in the values of the constants appearing in that equation and find out at what height the pressure is 10% of its value at sea level. At what height is it 1%?

b) What is the density of air at this height, compared to the density at sea level.

c) What is the air pressure at the top of mount Everest and at the height $40,000$ feet, where most of the air planes fly? Why the cabins of air planes have to be pressurized? The cabins are usually pressurized to only $0.85atm$. What is the force per unit area on the cabin walls pushing outward? Any idea why don't they pressurize the cabins to $1atm$ to make it more comfortable (no popping of ears)?

3. ## Work on a Van-der Wall's Gas

The Van-der Wall's equation of state:

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT,$$

where n is number of moles and a, b and R are constants, is a better approximation to the behavior of real gasses compared to the ideal gas equation of state.

For a Van-der Wall's gas, calculate the work done in compressing the gas "isothermally" from V_1 to V_2, $V_2 < V_1$.

4. **A Photon Gas**

The equation of state for electromagnetic field in a box (photon gas) is,

$$P = \frac{1}{3}\sigma T^4, \text{where } \sigma \text{ is a constant.}$$

Find the work done in taking this box from (V_1, T_1) state to (V_2, T_2) state along the two paths a and b in the figure below.